

Robert L. McDonald, Chapter 9: "Parity and Other Option Relationships"
in *Derivatives Markets*, Second Edition, pp. 281–309.

OUTLINE

I. PUT-CALL PARITY

A. Overview

1. Optionality involves being able to avoid unprofitable transactions
2. **Synthetic forward** - forward "created by buying the call and selling the put"
3. Put-call parity

$$\text{Call} - \text{Put} = \text{PV}(\text{Forward Price} - \text{Strike Price})$$

$$C(K, T) - P(K, T) = \text{PV}_{0,T}(F_{0,T} - K) = e^{-rT}(F_{0,T} - K), \text{ where}$$

$C(K, T)$ - price of European call with strike price K and time to expiration T

$P(K, T)$ - price of European put with strike price K and time to expiration T

$e^{-rT}F_{0,T}$ - prepaid forward price for the asset

$e^{-rT}K$ - prepaid forward price for the strike

4. $\text{PV}_{0,T}(F_{0,T} - K)$ is premium to pay for creating synthetic long forward position at a price lower than the forward price
5. Parity generally does not apply for American options

B. Options on Stocks

1. European option on a stock

$$C(K, T) = P(K, T) + [S_0 - \text{PV}_{0,T}(\text{Div})] - e^{-rT}K, \text{ where}$$

S_0 - current stock price

Div - stream of dividends paid on the stock

2. European option on an index

$$C(K, T) = P(K, T) + S_0e^{-\delta T} - \text{PV}_{0,T}(K)$$

3. Option premiums on an at-the-money European call and an at-the-money European put differ
 - a. If buy call and sell put, difference reflects interest on the deferral of payment for the stock
 - b. If sell call and buy put, difference reflects compensation for deferring receipt of the stock price
 - c. False explanation for the difference is the unlimited profit on the call and limited profit on the put
4. Differences between outright purchase of a stock and synthetic position
 - a. Synthetic does not pay dividends

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- b. Synthetic does not have voting rights

5. Synthetic stock

$$S_0 = C(K, T) - P(K, T) + PV_{0,T}(\text{Div}) + e^{-rT}K$$

- a. Differs from purchase of stock by timing differences
- b. If dividend paying, need to lend the PV of dividends

6. Synthetic T-bills

$$S_0 + P(K, T) - C(K, T) = PV_{0,T}(K) + PV_{0,T}(\text{Div}), \text{ where}$$

$PV(K) + PV_{0,T}(\text{Div})$ - cost

$K + FV_{0,T}(\text{Div})$ - payment at expiration

- a. Taxed as interest
- b. **Conversion** - "risk-free position consisting of an asset, a purchased put, and a written call"
- c. **Reverse conversion** - "short position in an asset coupled with a purchased call and written put, both with the same strike price and time to expiration"
- d. Reverse conversion equivalent to a short bond

7. Synthetic options

- a. Equivalence of a call and a leveraged position on the underlying asset insured by the purchase of a put

$$C(K, T) = S_0 - PV_{0,T}(\text{Div}) - PV_{0,T}(K) + P(K, T)$$

- b. Equivalence of a put and a short position on the stock insured by the purchase of a call

$$P(K, T) = C(K, T) - S_0 + PV_{0,T}(K) + PV_{0,T}(\text{Div})$$

C. Options on Currencies and Bonds

1. Option to buy one euro by paying x_0

$$C(K, T) - P(K, T) = x_0 e^{-r_\epsilon T} - K e^{-rT}, \text{ where}$$

x_0 - current exchange rate denominated as $\$/\epsilon$

r_ϵ - euro-denominated interest rate

r - dollar-denominated interest rate

2. Option on a bond

$$C(K, T) = P(K, T) + [B_0 - PV_{0,T}(\text{Coupons})] - PV_{0,T}(K)$$

- a. Prepaid forward for a bond differs from bond price because of coupon payments
- b. If noncoupon bond, relationship is the same as that for a non-dividend-paying stock

PAST CAS AND SOA EXAMINATION QUESTIONSA. Put-Call Parity

A1. For a certain stock, the price of a put option with an exercise price of \$100 exercisable in one year is \$25. The current stock price is \$120. The cost to borrow money is 10% for one year. What is the price of a call with an exercise price of \$100 exercisable in one year?

A. < \$50 B. ≥ \$50 but < \$55 C. ≥ \$55 but < \$60 D. ≥ \$60 but < \$65 E. ≥ \$65
(92-5B-64-1)

A2. An investor would like to sell short ABC company's stock, currently valued at \$100, and will close out this position in one year. Assume the investor does not want to lose more than \$100 on this transaction when she closes out her position at the end of the year. Further assume a European options market exists for ABC stock. Given the following table, what is the expected cost of this insurance? Assume a risk-free interest rate of 10% per year, and ignore transaction costs and potential broker margin calls.

Term: One-Year <u>Strike Price</u>	Value of <u>Put</u>	
50	5	
75	10	
100	15	
125	30	
150	50	
175	72	
200	95	(94S-5B-29-1/1/1.5)

A3. [Buy call, sell put] = [Buy share, borrow exercise price] (95F-5B-10-MC)

A4. According to McDonald, as a stock price becomes very large, the call option value approaches the difference between the stock price and the exercise price. (95F-5B-11-MC)

A5. You purchased a share of XYZ Corporation at \$25 and it has now increased to \$45. The annual risk-free rate is 5% and the price for a three-month call with a \$35 exercise price is \$13. Assume you want to lock in a sale price of at least \$35 for the next three months. Assuming XYZ does not pay dividends, what is the cost of the option that achieves this result?

A. < \$2 B. ≥ \$2 but < \$3 C. ≥ \$3 but < \$4 D. ≥ \$4 but < \$5 E. ≥ \$5 (96S-5B-14-1)

A6. An increase in the price of the underlying common stock will increase the price of a put option on that stock, other things being held equal. (98S-5B-13-MC)

A7. You sell a one-year European call option on Greystokes Inc. with an exercise price of 110 and buy a one-year European put option with the same exercise price and term. The current risk-free rate is 12% and the value of your combined position is zero. Greystokes Inc. is a non-dividend-paying stock. What is the price of a share of Greystokes Inc.?

A. < 95.00 B. ≥ 95.00 but < 97.50 C. ≥ 97.50 but < 100.00 D. ≥ 100.00 but < 102.50
E. ≥ 102.50 (99S-5B-1-1)

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- A8. You purchased a share of XYZ Corporation at \$25 and it has now increased to \$45. The annual risk-free rate is 5% and the price for a three-month call with a \$35 exercise price is \$13. Assume you want to lock in a sale price of at least \$35 for the next three months. Assuming XYZ does not pay dividends, what is the cost of the option that achieves this result?
- A. 1.58 B. 2.58 C. 3.58 D. 4.58 E. 5.58 (Sample2-2-44)
- A9. As the stock price rises, the value of a put option falls. (00S-2-42-MC)

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A1. $C(K, T) = P(K, T) + S_0 - Ke^{-rT} = 25 + 120 - 100/1.10 = 54.09$, p. 305.

Answer: B

A2. The seller sells a share he does not own for \$100 for delivery in the future. He needs to buy a call with an exercise price of \$200 to protect him against a loss more than \$100.

$$C(K, T) = P(K, T) + S_0 - Ke^{-rT} = 95 + 100 - 200/1.10 = 13.18, \text{ p. 305.}$$

A3. F, p. 305 – Substitute "present value of exercise price" for "exercise price."

A4. F, p. 282 – Substitute "present value of the exercise price" for "exercise price."

A5. $Ke^{-rT} = 35/(1.05)^{25} = 34.58$

$$P(K, T) = C(K, T) + Ke^{-rT} - S_0 = 13 + 34.58 - 45 = 2.58, \text{ p. 305.}$$

Answer: B

A6. F, p. 285 – Substitute "A decrease" for "An increase."

A7. $S_0 = (C(K, T) - p) + Ke^{-rT} = 0 + 110/1.12 = 98.21$, p. 305.

Answer: C

A8. $Ke^{-rT} = 35/(1.05)^{25} = 34.58$

$$P(K, T) = C(K, T) + Ke^{-rT} - S_0 = 13 + 34.58 - 45 = 2.58, \text{ p. 305.}$$

Answer: B

A9. T, p. 285 – They are inversely related.