KPW 15: PAST CAS AND SoA EXAMINATION QUESTIONS

A. Method-of-Moments Estimation

A1. The survival times of 10 persons, each age 74 at diagnosis of a disease were .5, 1.3, 1.4, 1.6, 1.7, 2.2, 2.9, 3.8, 3.9, and 4.2 years. Using the method of moments, a gamma distribution with location parameter $\tau = 0$ and shape parameter $\alpha = 2$ is fit to the data. Determine the scale parameter β.

A. 1.000 B. 1.175 C. 1.350 D. 2.000 E. 2.350 (86S–5–A3)

A2. $X_1, X_2, \ldots, X_n$ is an independent sample drawn from a lognormal distribution with parameters μ and σ^2. You are given the following formulas:

$$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i \qquad S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2$$

In terms of $\bar{X}$ and S^2 obtain estimators for μ and σ^2 using the method of moments.

A. $\hat{\mu} = \bar{X}$; $\hat{\sigma}^2 = S^2/\bar{X}$ B. $\hat{\mu} = \ln \bar{X}$; $\hat{\sigma}^2 = \ln S^2$ C. $\hat{\mu} = \bar{X}$; $\hat{\sigma}^2 = \ln(S^2 + \bar{X}^2)$

D. $\hat{\mu} = .5 \ln(\bar{X}^3/[\bar{X} + S^2])$; $\hat{\sigma}^2 = \ln(S^2 - \bar{X}^2)$

E. $\hat{\mu} = \ln \bar{X} - .5(S^2/\bar{X}^2 + 1)$ $\hat{\sigma}^2 = \ln(S^2/\bar{X}^2 + 1)$ (86–4–55–2)

A3. A random sample of death records yields the following exact ages at death: 30, 50, 60, 60, 70, 90. The age at death of the population from which the sample is drawn follows a gamma distribution given by

$$f(x) = \frac{x^{\alpha-1} e^{-x/\beta}}{\beta^{\alpha}\,\Gamma(\alpha)}, \quad x > 0$$

The parameters α and β are estimated using the method of moments. Determine the estimate of α.

A. 6.0 B. 7.2 C. 9.0 D. 10.8 E. 12.2 (86F–160–12)

A4. Using the method of moments, estimate the mean of a lognormal distribution, given the sample:

3 4.5 6 6.25 7 7.5 8.5 10

A. < 6 B. ≥ 6 but < 6.25 C. ≥ 6.25 but < 6.5 D. ≥ 6.5 but < 6.75 E. ≥ 6.75 (87–4–60–1)

A5. You are given:

i) Five lives are observed from time $t = 0$ until death.
ii) Deaths occur at $t = 3, 4, 4, 11$, and 18.

Assume the lives are subject to the probability density function:

$$f(t) = \frac{te^{-t/c}}{c^2}, \quad t > 0$$

Determine c by the method of moments.

A. 1/4 B. 1/2 C. 1 D. 2 E. 4 (89S–160–14)

KPW 15

Solutions are based on pp. 375–77, 422–30 plus the pages cited.

A1. $\bar{x} = (.5 + 1.3 + 1.4 + 1.6 + 1.7 + 2.2 + 2.9 + 3.8 + 3.9 + 4.2)/10 = 2.35$

$2.35 = \bar{x} = \alpha\beta = 2\beta \qquad \hat{\beta} = 1.175$

Answer: B

A2. $\bar{X} = \exp[\mu + \sigma^2/2] \qquad \hat{\mu} = \ln \bar{X} - \sigma^2/2$

$S^2 + \bar{X}^2 = E[X^2] = \exp[2\mu + 2\sigma^2]$

$\ln(S^2 + \bar{X}^2) = 2\mu + 2\sigma^2 = (2)(\ln \bar{X} - \sigma^2/2) + 2\sigma^2 = \ln \bar{X}^2 + \sigma^2$

$\hat{\sigma}^2 = \ln(S^2/\bar{X}^2 + 1) \qquad \hat{\mu} = \ln \bar{X} - \sigma^2/2 = \ln \bar{X} - .5\ln(S^2/\bar{X}^2 + 1)$, p. 678.

Answer: E

A3. $\bar{x} = (30 + 50 + 60 + 60 + 70 + 90)/6 = 60$

$s^2 = [(30 - 60)^2 + (50 - 60)^2 + (2)(60 - 60)^2 + (70 - 60)^2 + (90 - 60)^2]/5 = 400$

$\hat{\alpha} = \bar{x}^2/s^2 = (60)^2/400 = 9$, p. 674.

Answer: C

A4. $\bar{x} = (3 + 4.5 + 6 + 6.25 + 7 + 7.5 + 8.5 + 10)/8 = 6.59$

Answer: D

A5. This is a gamma function with $\alpha = 2$.

$\bar{x} = (3 + 4 + 4 + 11 + 18)/5 = 8$

$8 = \bar{x} = \alpha c = 2c \qquad \hat{c} = 4$, p. 674.

Answer: E