MULTIPLE DECREMENTS: PAST CAS AND SoA EXAMINATION QUESTIONS

A1. In a double-decrement table associated with a certain body of lives, the two forces of decrement are each equal to .5. What is the impact on the probability of decrement from one cause if the force of decrement from the other cause is doubled?

$e^{-.5} = .61 \qquad e^{-1} = .37 \qquad e^{-1.5} = .22$

A. A decrease > .05 B. A decrease < .05 C. No change D. An increase < .05 E. An increase > .05 (82–4–30–2)

A2. A multiple-decrement table has two causes of decrement: (1) accident; (2) other than accident. You are given:

$\mu_y^{(1)}(t) = A,\ A > 0 \qquad \mu_y^{(2)}(t) = Bc^y,\ \text{ where } B > 0,\ c > 0$

Determine the probability of death by accident for (x).

A. $A/\mathring{e}_x$ B. $\mathring{e}_x/A$ C. $A\mathring{e}_x$ D. $1/A\mathring{e}_x$ E. $1 - A/\mathring{e}_x$ (87S–150–30)

A3. You are given the following information for a person aged x:

$\mu_x^{(1)}(t) = .01 \qquad \mu_x^{(2)}(t) = .04,\ \text{ where } t \geq 0,$

where the index (1) indicates death due to accidental causes and the index (2) indicates death due to nonaccidental causes. What is the probability that (x) will die within the next 10 years due to nonaccidental causes?

A. < .30 B. ≥ .30 but < .31 C. ≥ .31 but < .32 D. ≥ .32 but < .33 E. ≥ .33 (90–4–19–2)

A4. A multiple-decrement table has two causes of decrement: 1) accident and 2) other than accident. You are given:

$\mu_y^{(1)}(t) = .0010 \qquad \mu_y^{(2)}(t) = (.0005)(10)^{.05y}$

Determine the probability of death by accident for (x) in terms of $\mathring{e}_x$.

A. $.0005\mathring{e}_x$ B. $.0010\mathring{e}_x$ C. $.0050\mathring{e}_x$ D. $.0100\mathring{e}_x$ E. $.0500\mathring{e}_x$ (90S–150–30)

A5. For a triple-decrement table, you are given:

i) Each decrement has a constant force of decrement over each year of age.
ii) The following table of values:

j	1	2	3
$\mu_x^{(j)}(t)$	.2	.4	.6

Calculate $q_x^{(2)}$.

A. .20 B. .23 C. .26 D. .30 E. .33 (90F–150–16)

Multiple Decrements

Solutions are based on Cunningham, pp. 342–43, 345–49, and the pages from Bowers listed below.

A1. $q_x^{(1)} = \int_0^1 {}_tp_x^{(\tau)} \mu_x^{(1)}(t)\,dt = .5\int_0^1 e^{-(.5+.5)t}\,dt) = .5e^{-1}\Big|_0^1 = (.5)(1 - e^{-1}) = .315$

$q'^{(1)}_x = \int_0^1 {}_tp'^{(\tau)}_x \mu_x^{(1)}(t)\,dt = .5\int_0^1 e^{-(.5+1)t}\,dt = (.5/1.5)e^{-1}\Big|_0^1 = (1 - e^{-1.5})/3 = .26$

$q'^{(1)}_x - q'^{(1)}_x = .26 - .315 = -.055$, p. 312.

Answer: A

A2. ${}_\infty q_x^{(1)} = \int_0^\infty {}_tp_x^{(\tau)} \mu_x^{(1)}(t)\,dt = A\int_0^\infty {}_tp_x^{(\tau)}\,dt = A\,\mathring{e}_x$, pp. 68, 312.

Answer: C

A3. ${}_{10}q_x^{(2)} = \int_0^{10} {}_{10}p_x^{(\tau)} \mu_x^{(2)}(t)\,dt = .04\int_0^{10} e^{-(.01+.04)t}\,dt = (.04/.05)e^{-.05t}\Big|_0^1 = (.8)(1 - e^{-.5}) = .315$, p. 312.

Answer: C

A4. See A2. ${}_\infty q_x^{(1)} = .0010\mathring{e}_x$

Answer: B

A5. $q_x^{(2)} = \int_0^1 {}_tp_x^{(\tau)} \mu_x^{(2)}(t)\,dt = \int_0^\infty e^{-(.2+.4+.6)t}(.4)\,dt = (-.4/1.2)\,e^{-1.2t}\Big|_0^1 = (1 - e^{-1.2})/3 = .23$, p. 312.

Answer: B