Robert L. McDonald, Chapter 9: "Parity and Other Option Relationships"
in *Derivatives Markets*, Second Edition, pp. 281–309.

OUTLINE

I. PUT-CALL PARITY

A. Overview

1. Optionality involves being able to avoid unprofitable transactions
2. **Synthetic forward** - forward "created by buying the call and selling the put"
3. Put-call parity

Call – Put = PV(Forward Price – Strike Price)

$C(K, T) - P(K, T) = PV_{0,T}(F_{0,T} - K) = e^{-rT}(F_{0,T} - K)$, where

C(K, T) - price of European call with strike price K and time to expiration T
P(K, T) - price of European put with strike price K and time to expiration T
$e^{-rT}F_{0,T}$ - prepaid forward price for the asset
$e^{-rT}K$ - prepaid forward price for the strike

4. $PV_{0,T}(F_{0,T} - K)$ is premium to pay for creating synthetic long forward position at a price lower than the forward price
5. Parity generally does not apply for American options

B. Options on Stocks

1. European option on a stock

$C(K, T) = P(K, T) + [S_0 - PV_{0,T}(Div)] - e^{-rT}K$, where

S_0 - current stock price
Div - stream of dividends paid on the stock

2. European option on an index

$C(K, T) = P(K, T) + S_0e^{-\delta T} - PV_{0,T}(K)$

3. Option premiums on an at-the-money European call and an at-the-money European put differ
   a. If buy call and sell put, difference reflects interest on the deferral of payment for the stock
   b. If sell call and buy put, difference reflects compensation for deferring receipt of the stock price
   c. False explanation for the difference is the unlimited profit on the call and limited profit on the put
4. Differences between outright purchase of a stock and synthetic position
   a. Synthetic does not pay dividends
   b. Synthetic does not have voting rights

5. Synthetic stock

$$S_0 = C(K, T) - P(K, T) + PV_{0,T}(Div) + e^{-rT}K$$

a. Differs from purchase of stock by timing differences
b. If dividend paying, need to lend the PV of dividends

6. Synthetic T-bills

$$S_0 + P(K, T) - C(K, T) = PV_{0,T}(K) + PV_{0,T}(Div), \text{ where}$$

$PV(K) + PV_{0,T}(Div)$ - cost
$K + FV_{0,T}(Div)$ - payment at expiration

a. Taxed as interest
b. **Conversion** – "risk-free position consisting of an asset, a purchased put, and a written call"
c. **Reverse conversion** – "short position in an asset coupled with a purchased call and written put, both with the same strike price and time to expiration"
d. Reverse conversion equivalent to a short bond

7. Synthetic options

a. Equivalence of a call and a leveraged position on the underlying asset insured by the purchase of a put

$$C(K, T) = S_0 - PV_{0,T}(Div) - PV_{0,T}(K) + P(K, T)$$

b. Equivalence of a put and a short position on the stock insured by the purchase of a call

$$P(K, T) = C(K, T) - S_0 + PV_{0,T}(K) + PV_{0,T}(Div)$$

C. <u>Options on Currencies and Bonds</u>

1. Option to buy one euro by paying x_0

$$C(K, T) - P(K, T) = x_0 e^{-r\varepsilon} - Ke^{-rT}, \text{ where}$$

x_0 - current exchange rate denominated as \$/ε
r_ε - euro-denominated interest rate
r - dollar-denominated interest rate

2. Option on a bond

$$C(K, T) = P(K, T) + [B_0 - PV_{0,T}(Coupons)] - PV_{0,T}(K)$$

a. Prepaid forward for a bond differs from bond price because of coupon payments
b. If noncoupon bond, relationship is the same as that for a non-dividend-paying stock

PAST CAS AND SoA EXAMINATION QUESTIONS

A. Put-Call Parity

A1. For a certain stock, the price of a put option with an exercise price of $100 exercisable in one year is $25. The current stock price is $120. The cost to borrow money is 10% for one year. What is the price of a call with an exercise price of $100 exercisable in one year?

A. < $50 B. ≥ $50 but < $55 C. ≥ $55 but < $60 D. ≥ $60 but < $65 E. ≥ $65 (92–5B–64–1)

A2. An investor would like to sell short ABC company's stock, currently valued at $100, and will close out this position in one year. Assume the investor does not want to lose more than $100 on this transaction when she closes out her position at the end of the year. Further assume a European options market exists for ABC stock. Given the following table, what is the expected cost of this insurance? Assume a risk-free interest rate of 10% per year, and ignore transaction costs and potential broker margin calls.

Term: One-Year Strike Price	Value of Put
50	5
75	10
100	15
125	30
150	50
175	72
200	95

(94S–5B–29–1/1/1.5)

A3. [Buy call, sell put] = [Buy share, borrow exercise price] (95F–5B–10–MC)

A4. According to McDonald, as a stock price becomes very large, the call option value approaches the difference between the stock price and the exercise price. (95F–5B–11–MC)

A5. You purchased a share of XYZ Corporation at $25 and it has now increased to $45. The annual risk-free rate is 5% and the price for a three-month call with a $35 exercise price is $13. Assume you want to lock in a sale price of at least $35 for the next three months. Assuming XYZ does not pay dividends, what is the cost of the option that achieves this result?

A. < $2 B. ≥ $2 but < $3 C. ≥ $3 but < $4 D. ≥ $4 but < $5 E. ≥ $5 (96S–5B–14–1)

A6 An increase in the price of the underlying common stock will increase the price of a put option on that stock, other things being held equal. (98S–5B–13–MC)

A7. You sell a one-year European call option on Greystokes Inc. with an exercise price of 110 and buy a one-year European put option with the same exercise price and term. The current risk-free rate is 12% and the value of your combined position is zero. Greystokes Inc. is a non-dividend-paying stock. What is the price of a share of Greystokes Inc.?

A. < 95.00 B. ≥ 95.00 but < 97.50 C. ≥ 97.50 but < 100.00 D. ≥ 100.00 but < 102.50 E. ≥ 102.50 (99S–5B–1–1)

A8. You purchased a share of XYZ Corporation at $25 and it has now increased to $45. The annual risk-free rate is 5% and the price for a three-month call with a $35 exercise price is $13. Assume you want to lock in a sale price of at least $35 for the next three months. Assuming XYZ does not pay dividends, what is the cost of the option that achieves this result?

A. 1.58 B. 2.58 C. 3.58 D. 4.58 E. 5.58 (Sample2–2–44)

A9. As the stock price rises, the value of a put option falls. (00S–2–42–MC)

A1. $C(K, T) = P(K, T) + S_0 - Ke^{-rT} = 25 + 120 - 100/1.10 = 54.09$, p. 305.

Answer: B

A2. The seller sells a share he does not own for \$100 for delivery in the future. He needs to buy a call with an exercise price of \$200 to protect him against a loss more than \$100.

$C(K, T) = P(K, T) + S_0 - Ke^{-rT} = 95 + 100 - 200/1.10 = 13.18$, p. 305.

A3. F, p. 305 – Substitute "present value of exercise price" for "exercise price."

A4. F, p. 282 – Substitute "present value of the exercise price" for "exercise price."

A5. $Ke^{-rT} = 35/(1.05)^{.25} = 34.58$

$P(K, T) = C(K, T) + Ke^{-rT} - S_0 = 13 + 34.58 - 45 = 2.58$, p. 305.

Answer: B

A6. F, p. 285 – Substitute "A decrease" for "An increase."

A7. $S_0 = (C(K, T) - p) + Ke^{-rT} = 0 + 110/1.12 = 98.21$, p. 305.

Answer: C

A8. $Ke^{-rT} = 35/(1.05)^{.25} = 34.58$

$P(K, T) = C(K, T) + Ke^{-rT} - S_0 = 13 + 34.58 - 45 = 2.58$, p. 305.

Answer: B

A9. T, p. 285 – They are inversely related.