

BODIE 7: PAST CAS AND SoA EXAMINATION QUESTIONSB. Portfolio of Two Risky Assets: Expected Returns and Variances

B1. You are given the following:

<u>Stock</u>	<u>Variance</u>	<u>Stock to Stock Correlation Coefficients</u>		<u>Weight Held</u>
		<u>#1</u>	<u>#2</u>	
#1	.09	1.0	-.5	.6
#2	.16	-.5	1.0	.4

Determine the expected risk for the portfolio as measured by its standard deviation.

A. .0036 B. .0292 C. .0596 D. .1108 E. .1709 (88-220-18)

B2. You are given the following data about a portfolio consisting of two stocks:

	<u>Stock A</u>	<u>Stock B</u>
Percent of portfolio invested	50%	50%
Variance of return on stock	.09	.04

Covariance of returns on stocks A and B is .05. The portfolio variance is:

A. $< .05$ B. $\geq .05$ but $< .06$ C. $\geq .06$ but $< .07$ D. $\geq .07$ but $< .08$ E. $\geq .08$
(91-5B-56-1) (Sample1-2-34)

B3. You are given a portfolio consisting of two stocks with equal market value. The rate of return of stock 1 has a variance of .09. The rate of return of stock 2 has a variance of .05. The correlation coefficient between the rates of return of the two stocks is .3. What is the standard deviation of the portfolio rate of return?

A. $< 20\%$ B. $\geq 20\%$ but $< 30\%$ C. $\geq 30\%$ but $< 40\%$ D. $\geq 40\%$ but $< 50\%$ E. $\geq 50\%$
(92-5B-58-1)

B4. Your broker has identified a \$10,000 investment in a small company growth stock. The broker indicates that you could lose 25% or make 50% on this investment in a year. The broker has offered to protect you from a loss on this stock for an additional \$1,212. You want to minimize your brokerage commissions but still invest in this stock and receive the same returns as offered above. What combination of stock purchase and investment at a 10% rate will guarantee the same return after one year as offered by the broker? (92-5B-83-2)

B5. If a portfolio consists of two stocks, the expected rate of return on the portfolio will be at least as high as the lower of the expected returns of the two stock. (92-220-25-MC)

B6. Assume you currently own 1,000 shares of stock in XYZ company, which is currently trading at \$20 per share. The correlation coefficients between XYZ and other stocks are given in the table below. Assume that you have \$30,000 in additional funds to invest, that you intend to invest that entire amount in one of the stocks listed below (including XYZ), and that you will not sell your original 1,000 shares of XYZ stock. Which one stock should you purchase if you wish to minimize the total risk of your portfolio? Show all calculations.

<u>Stock</u>	<u>Standard Deviation</u>	<u>Correlation Coefficient with XYZ Stock</u>	
ABC	.40	0.20	
DEF	.30	0.40	
GHI	.40	0.50	
XYZ	.50	1.00	(94S-5B-26-2.5)

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B1. $\sigma_P^2 = w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2w_1w_2 \text{Cov}(r_1, r_2) = (.6)^2(.09) + (.4)^2(.16) + 2(.6)(.4)(-.5)(.3)(.4)$

$\sigma_P^2 = .0292 \quad \sigma_P = \sqrt{.0292} = .1709$, p. 210.

Answer: E

B2. $\sigma_P^2 = w_A^2 \sigma_A^2 + w_B^2 \sigma_B^2 + 2w_Aw_B \text{Cov}(r_A, r_B)$

$\sigma_P^2 = (.5)^2(.09) + (.5)^2(.04) + (2)(.5)(.5)(.05) = .0575$, p. 210.

Answer: B

B3. $\sigma_P^2 = w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2w_1w_2 \text{Cov}(r_1, r_2)$

$\sigma_P^2 = (.5)^2(.09) + (.5)^2(.05) + (2)(.5)(.5)(.3)(\sqrt{.09})(\sqrt{.05}) = .045$

$\sigma_P = \sqrt{.045} = 21.2\%$, p. 210.

Answer: B

B4. There are two outcomes that need to be duplicated. Outcome #1 is where a loss occurs but insurance fully indemnifies the investor; i.e., there is no change in the original investment. Outcome #2 is where the investment experiences a 50% gain; i.e., the investment reaches 15,000.

- 1) Set up two simultaneous equations to equal the two outcomes where X is the amount of stock purchased and Y is the amount invested that will earn 10%:

Outcome #1: $X(.75) + Y(1.1) = 10,000$

Outcome #2: $X(1.5) + Y(1.1) = 15,000$

- 2) Solve the equations:

$X = 6,667 \quad Y = 4,545 \quad X + Y = 11,212$, p. 208.

B5. T, p. 208 – The portfolio expected return is a weighted average of the individual returns.

B6. Calculate the variance of each portfolio, with 40% (20,000) invested in XYZ and 60% (30,000) invested in the other stock:

$\sigma_P^2 = w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2w_1w_2 \text{Cov}(r_1, r_2)$

$\sigma_{XYZ/ABC}^2 = (.4)^2(.5)^2 + (.6)^2(.4)^2 + (2)(.4)(.6)(.2)(.5)(.4) = .1168$

$\sigma_{XYZ/DEF}^2 = (.4)^2(.5)^2 + (.6)^2(.3)^2 + (2)(.4)(.6)(.4)(.5)(.3) = .1012$

$\sigma_{XYZ/GHI}^2 = (.4)^2(.5)^2 + (.6)^2(.4)^2 + (2)(.4)(.6)(.5)(.5)(.4) = .1456$

Risk is thus minimized by investing in stock DEF as this produces the portfolio with the least variance, p. 210.