INSURANCES/ANNUITIES: PAST CAS AND SoA EXAMINATION QUESTIONS

A. Multilife Survival Statuses

A1. Smith, age 65, and Jones, age 70, purchase a $1,000 joint one-year endowment insurance policy. Death benefits are paid at the moment of death. The nominal annual interest rate is 10% compounded semimonthly. You are given that $\ddot{a}_{65:70:\overline{2|}} = 1.8345$. In which of the following ranges is the actuarial present value for this policy? (Use standard approximations.)

A. < $910 B. ≥ $910 but < $920 C. ≥ $920 but < $930 D. ≥ $930 but < $940 E. ≥ $940
(83F–4–35)

A2. You are given:

$\ddot{a}_{xy} = 10 \qquad {}^2\ddot{a}_{xy} = 7 \qquad Var(\ddot{a}_{\overline{K+1|}}) = 27$

K is the number of years completed to the failure of the status (xy). Calculate the discount rate d.

A. 3/127 B. 1/20 C. 6/127 D. 1/10 E. 13/127 (84S–4–C3)

A3. Find $\ddot{a}_{xy:\overline{3|}}$ at i = 5%. Use the following mortality tables:

t	q_{x+t}	q_{y+t}
0	.1	.2
1	.2	.3
2	.3	.4
3	.4	.5

A. < 2.00 B. ≥ 2.00 but < 2.10 C. ≥ 2.10 but < 2.20 D. ≥ 2.20 but < 2.30 E. ≥ 2.30
(86–4–28–1)

A4. If mortality for each life (x) and (y) follows de Moivre's law with $\omega = 100$, $\bar{A}_{xy} = (i/\delta)A_{xy}$. (86F–150–A16–MC)

A5. An insurance benefit pays 1 at the later of n years or the failure of the status $\overline{xy}$. Which of the following correctly expresses the actuarial present value for this benefit:

1. $v^n {}_nq_{\overline{xy}} + v^n {}_np_{xy} \bar{A}_{\overline{x+n:y+n}}$ 2. $\bar{A}_{\overline{xy}} - \bar{A}_{\overline{xy}:\overline{n|}}$

3. $v^n({}_np_x \bar{A}_{x+n} + {}_np_y \bar{A}_{y+n} - {}_np_{xy} \bar{A}_{x+n:y+n} + {}_nq_{\overline{xy}})$

A. None of these expressions correctly state the value. B. 1 C. 2 D. 3
E. None of these answers are correct. (87F–150–28)

A6. Assuming that the time-until-death random variables for (x) and (y) are independent, $A_{xy} + A_{\overline{xy}} = A_x + A_y$. (88–4–26–MC)

A7. Z is the actuarial present value random variable for a discrete whole life insurance of 1 issued to (x) and (y) that pays 1 at the first death and 1 at the second death. You are given:

$a_x = 9 \qquad a_y = 13 \qquad i = .04$

Calculate E[Z].

A. .08 B. .28 C. .69 D. 1.08 E. 1.15 (89S–150–30)

Insurance/Annuities

Solutions are based on Cunningham, pp. 312–20, 331, and the pages from Bowers listed below.

A1. $1 + i = (1.05)^2 = 1.1025 \quad \log 1.1025 = .097580$

$$p_{65:70} = \frac{\ddot{a}_{65:70:\overline{2|}} - 1}{v} = \frac{1.8345 - 1}{1/1.1025} = .92$$

$$\bar{A}_{65:70:\overline{2|}} = \frac{i}{\delta}\bar{A}^{\;1}_{65:70:\overline{2|}} + A_{65:70:\overline{2|}}^{\quad\;\;1} = \frac{.1025}{.097580} v(1 - p_{65:70}) + vp_{65:70}$$

$1{,}000\bar{A}_{65:70:\overline{2|}} = [1{,}000][(1.05042)(1.1025)^{-1}(1 - .92) + (1.1025)^{-1}(.920)] = 910.69$, pp. 120, 280.

Answer: B

A2. $$d = \frac{2(\ddot{a}_{xy} - {}^2\ddot{a}_{xy})}{\mathrm{Var}(\ddot{a}_{\overline{K+1|}}) + (\ddot{a}_{xy})^2 - {}^2\ddot{a}_{xy}} = \frac{(2)(10 - 7)}{27 + (10)^2 - 7} = 1/20$$, pp. 164, 280.

Answer: B

A3. $p_{xy} = p_x\, p_y = (1 - .1)(1 - .2) = .72$

$_2p_{xy} = p_{xy}\, p_{x+1}\, p_{y+1} = (.72)(1 - .2)(1 - .3) = .4032$

$\ddot{a}_{xy} = 1 + vp_{xy} + v^2\, {}_2p_{xy}$

$\ddot{a}_{xy} = 1 + .72/1.05 + .4032/(1.05)^2 = 2.05$, pp. 280–81.

Answer: B

A4. F, p. 74, 120, 281 – The formula is that for the uniform distribution of deaths, not de Moivre's law.

A5.
1. F, pp. 280–81 – This does not provide premium for the coverage of the situation where one and only one of the lives dies before n.
2. F, pp. 280–81 – An exponent is required over $\overline{xy}$ in the second term.
3. T, pp. 280–81

Answer: D

A6. T, p. 281.

A7. $d = .04/1.04 = .03846 \qquad E[Z] = A_{xy} + A_{\overline{xy}} = A_x + A_y = 1 - d\ddot{a}_x + 1 - d\ddot{a}_y$

$E[Z] = 1 - (.03846)(9 + 1) + 1 - (.03846)(13 + 1) = 1.08$, pp. 144, 281.

Answer: D